The dynamical Casimir effect in braneworlds

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In braneworld cosmology the expanding Universe is realized as a brane moving through a warped higher-dimensional spacetime. Like a moving mirror causes the creation of photons out of vacuum fluctuations, a moving brane leads to graviton production. We show that, very generically, KK-particles scale like stiff matter with the expansion of the Universe and can therefore not represent the dark matter in a warped braneworld. We present results for the production of massless and Kaluza-Klein (KK) gravitons for bouncing branes in five-dimensional Anti de Sitter space. We find that for a realistic bounce the back reaction from the generated gravitons will be most likely relevant. This Letter summarizes the main results and conclusions from numerical simulations which are presented in detail in a long paper [M. Ruser and R. Durrer, Phys. Rev. D **76**, 104014 (2007), arXiv:0704.0790].

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Introduction: String theory, the most serious candidate for a quantum theory of gravity, predicts the existence of 'branes', i.e. hypersurfaces in the 10- (or 11-) dimensional spacetime on which ordinary matter, e.g. gauge particles and fermions, are confined. Gravitons can move freely in the 'bulk', the full higher dimensional spacetime [1].

The scenario, where our Universe moves through a five-dimensional Anti de Sitter (AdS) spacetime has been especially successful in reproducing the observed four-dimensional behavior of gravity. It has been shown that at sufficiently low energies and large scales, not only gravity on the brane looks four dimensional [2], but also cosmological expansion can be reproduced [3]. We shall concentrate here on this example and comment on behavior which may survive in other warped braneworlds.

We consider the following situation: A fixed 'static brane' is sitting in the bulk. The 'physical brane', our Universe, is first moving away from the AdS Cauchy horizon, approaching the second brane. This motion corresponds to a contracting Universe. After a closest encounter the physical brane turns around and moves away from the static brane. This motion mimics the observed expanding Universe.

The moving brane acts as a time-dependent boundary for the 5D bulk leading to production of gravitons from vacuum fluctuations in the same way a moving mirror causes photon creation from vacuum in dynamical cavities [4]. Apart from massless gravitons, braneworlds allow for a tower of Kaluza-Klein (KK) gravitons which appear as massive particles on the brane leading possibly to phenomenological consequences.

We postulate, that high energy stringy physics will lead to a turnaround of the brane motion, i.e., provoke a repulsion of the physical brane from the static one. This motion is modeled by a kink where the brane velocity changes sign. As we shall see, a perfect kink leads to divergent particle production due to its infinite acceleration. We therefore assume that the kink is rounded off at the string scale L_s. Then particles with energies $E > E_s = 1/L_s$ are not generated. This setup represents a regular 'bouncing Universe' as, for example the 'ekpyrotic Universe' [5]. Four-dimensional bouncing Universes have also been studied in Ref. [6].

Moving brane in AdS$_5$: Our starting point is the metric of AdS$_5$ in Poincaré coordinates:

$$ds^2 = g_{AB}dx^A dx^B = \frac{L^2}{y^2}\left[-dt^2 + \delta_{ij}dx^i dx^j + dy^2\right] \ . \tag{1}$$

The physical brane (our Universe) is located at some time-dependent position $y = y_b(t)$, while the static brane is at a fixed position $y = y_s > y_b(t)$. The scale factor on the brane is

$$a(\eta) = \frac{L}{y_b(t)}\,, \ \ d\eta = \sqrt{1-v^2}dt = \gamma^{-1}dt\ , \ \ v = \frac{dy_b}{dt}\ ,$$

where we have introduced the brane velocity v and the conformal time η on the brane. If $v \ll 1$, the junction conditions lead to the Friedmann equations on the brane. For reviews see [7, 8]. Defining the string and Planck scales by $\kappa_5 \equiv L_s^3$ and $\kappa_4 \equiv L_{Pl}^2$ the Randall-Sundrum (RS) fine tuning condition [2] implies

$$\frac{L}{L_s} = \left(\frac{L_s}{L_{Pl}}\right)^2 \ . \tag{2}$$

We assume that the brane energy density is dominated by a radiation component. The contracting ($t < 0$) and expanding ($t > 0$) phases are then described by

$$a(t) \ = \ \frac{|t| + t_b}{L}\,, \qquad y_b(t) = \frac{L^2}{|t|+t_b}\,, \tag{3}$$

$$v(t) \ = \ -\frac{\mathrm{sign}(t)L^2}{(|t|+t_b)^2} \simeq -HL \tag{4}$$

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where $H = (da/d\eta)/a^2$ is the Hubble parameter and we have used that $\eta \simeq t$ if $v \ll 1$. A small velocity also requires $y_b(t) \ll L$. The transition from contraction to expansion is approximated by a kink at $t = 0$, such that at the moment of the bounce

$$|v(0)| \equiv v_b = \frac{L^2}{t_b^2}\ ,\ \ a_b = a(0) = \frac{1}{\sqrt{v_b}}\ ,\ \ H_b^2 = \frac{v_b^2}{L^2}\ . \quad (5)$$

Tensor perturbations: We now consider tensor perturbations h_{ij} on this background,

$$ds^2 = \frac{L^2}{y^2}\left[-dt^2 + (\delta_{ij} + 2h_{ij})dx^i dx^j + dy^2\right]\ . \quad (6)$$

For each polarization, their amplitude h satisfies the Klein-Gordon equation in AdS$_5$ [8]

$$\left[\partial_t^2 + k^2 - \partial_y^2 + \frac{3}{y}\partial_y\right] h(t, y; \mathbf{k}) = 0 \quad (7)$$

where $k = |\mathbf{k}|$ is the momentum parallel to the brane and h is subject to the boundary (2nd junction) conditions

$$\left(v\partial_t + \partial_y\right) h|_{y_b(t)} = 0\ \rightarrow\ \partial_y h|_{y_b(t)} = 0\ \text{ and }\ \partial_y h|_{y_s} = 0\ . \quad (8)$$

Being interested in late-time (low energy) effects, we have approximated the first of those conditions by a Neumann condition ($v \ll 1$). Then, the spatial part of Eq. (7) together with (8) forms a Strum-Liouville problem at any given time and therefore has a complete orthonormal set of eigenfunctions $\{\phi_\alpha(t, y)\}_{\alpha=0}^{\infty}$. These 'instantaneous' mode functions are given by

$$\begin{aligned}\phi_0(t) &= \frac{y_s y_b(t)}{\sqrt{y_s^2 - y_b^2(t)}}. &(9)\\ \phi_n(t, y) &= N_n(t) y^2 C_2(m_n(t), y_b(t), y) \ \text{ with}\\ C_\nu(m, x, y) &= Y_1(mx)J_\nu(my) - J_1(mx)Y_\nu(my) &(10)\end{aligned}$$

and satisfy $[-\partial_y^2 + (3/y)\partial_y]\phi_\alpha(y) = m_\alpha^2\phi_\alpha(y)$ as well as (8). N_n is a time-dependent normalization condition. More details can be found in [9]. The massless mode ϕ_0 represents the ordinary four-dimensional graviton on the brane, while the massive modes are KK gravitons. Their masses are quantized by the boundary condition at the static brane which requires $C_1(m_n, y_b, y_s) = 0$. At late times and for large n the KK masses are roughly given by $m_n \simeq n\pi/y_s$. The gravity wave amplitude h may now be decomposed as [9]

$$h(t, y; \mathbf{k}) = \sqrt{\frac{\kappa_5}{L^3}}\sum_{\alpha=0}^{\infty} q_{\alpha,\mathbf{k}}(t)\phi_\alpha(t, y) \quad (11)$$

where the prefactor assures that the variables $q_{\alpha,\mathbf{k}}$ are canonically normalized. Their time evolution is determined by the brane motion [cf. Eq. (14)].

Localization of gravity: From the above expressions and using $L/y_b(t) = a(t)$, we can determine the late-time behavior of the mode functions ϕ_α on the brane ($y_b \ll L \ll y_s$)

$$\phi_0(t, y_b) \rightarrow \frac{L}{a}\ ,\ \ \phi_n(t, y_b) \rightarrow \frac{L^2}{a^2}\sqrt{\frac{\pi m_n}{2y_s}}\ . \quad (12)$$

At this point we can already make two crucial observations: First, the mass m_n is a comoving mass. The instantaneous energy of a KK graviton is $\omega_{n,k} = \sqrt{k^2 + m_n^2}$, where k denotes comoving wave number. The 'physical mass' of a KK mode measured by an observer on the brane with cosmic time $d\tau = a dt$ is therefore m_n/a, i.e. the KK masses are redshifted with the expansion of the Universe. This comes from the fact that m_n is the wave number corresponding to the y direction with respect to the bulk time t which corresponds to *conformal time* η on the brane and not to physical time. It implies that the energy of KK particles on a moving AdS brane is redshifted like that of massless particles. From this alone we would expect the energy density of KK modes on the brane decays like $1/a^4$.

But this is not all. In contrast to the zero mode which behaves as $\phi_0(t, y_b) \propto 1/a$ the KK-mode functions $\phi_n(t, y_b)$ decay as $1/a^2$ with the expansion of the Universe and scale like $1/\sqrt{y_s}$. Consequently the amplitude of the KK modes on the brane dilutes rapidly with the expansion of the Universe and is in general smaller the larger y_s. This can be understood by studying the probability of finding a KK-graviton at position y in the bulk which turns out to be much larger in regions of less warping than in the vicinity of the physical brane[9]. If KK gravitons are present on the brane, they escape rapidly into the bulk, i.e., the moving brane looses them, since their wave function is repulsed away from the brane. This causes the additional $1/a$-dependence of $\phi_n(t, y_b)$ compared to $\phi_0(t, y_b)$. The $1/\sqrt{y_s}$-dependence expresses the fact that the larger the bulk the smaller the probability to find a KK-graviton at the position of the moving brane. This behavior reflects the localization of gravity: traces of the five-dimensional nature of gravity like KK gravitons become less and less 'visible' on the brane as time evolves. As a consequence, the energy density of KK gravitons at late times on the brane behaves as

$$\rho_{\rm KK} \propto 1/a^6\ . \quad (13)$$

It means that KK gravitons redshift like stiff matter and cannot be the dark matter in an AdS braneworld since their energy density does not have the required $1/a^3$ behavior. They also do not behave like dark radiation [7, 8] as one might naively expect. This new result is derived in detail in Ref. [9]. It is based on the calculation of $\langle \dot{h}^2(t, y_b, \mathbf{k})\rangle \propto \langle \dot{q}_{\alpha,\mathbf{k}}^2(t)\rangle \phi_\alpha^2(t, y_b)$ where the bracket incorporates a quantum expectation value with respect to a well-defined initial vacuum state and averaging over several oscillations of the field [9]. An overdot denotes the derivative with respect to t. The scaling behaviour

(13) is due to $\phi_\alpha^2(t, y_b)$ only, since graviton production from vacuum fluctuations has ceased at late times (like in radiation domination) which is necessary for a meaningful particle definition. Then, $\langle \dot{q}^2_{\alpha,\mathbf{k}}(t)\rangle$ is related to the number of produced gravitons and is constant in time. In case that amplification of tensor perturbation is still ongoing, e.g., during a de Sitter phase, the energy density related to the massive modes might scale differently. The scaling behavior (13) remains valid also when the fixed brane is sent off to infinity and we end up with a single braneworld in AdS$_5$, like in the Randall-Sundrum II scenario [2]. The situation is not altered if we replace the graviton by a scalar or vector degree of freedom in the bulk. Since every bulk degree of freedom must satisfy the five-dimensional Klein-Gordon equation, the mode functions will always be the functions ϕ_α, and the energy density of the KK-modes decays like $1/a^6$. KK particles on a brane moving through an AdS bulk cannot play the role of dark matter.

It is important here that we consider a static bulk and the time depencence of the brane comes solely from its motion through the bulk. In Ref. [10] the situation of a *fixed brane* in a *time-dependent bulk* is discussed. There it is shown that under certain assumption (separability of the y and t dependence of fluctuations), the energy density of KK modes on a low energy cosmological brane does scale like $1/a^3$ which seems to be in contradiction with our result. However, the approximations used in [10] lead to a system of equations governing the expansion of the Universe but neglecting the time dependence of the bulk. The situation is then effectively four dimensional even for the KK modes; effects of the fifth dimension like the possibility of KK gravitons escaping into the bulk seem to be lost in this approach. In our case we would have a similar situation if we keep the expansion on the brane $a(t)$ but take the position of the brane in the bulk as static $y_b(t) = \text{const}$, which is not consistent with the general relation $y_b(t) = L/a(t)$. [For a fixed physical mass $M = m/a$, if we neglect the time dependence of $\phi_n(y_b(t)) \propto 1/a^2$ we also obtain an energy density for this mass proportional to $1/a^3$.]

Particle production: The equation of motion for the canonical variables $q_{\alpha,\mathbf{k}}$ is of the form, see Ref. [9],

$$\ddot{q}_{\alpha,\mathbf{k}} + \omega^2_{\alpha,k} q_{\alpha,\mathbf{k}} = \sum_{\beta\neq\alpha} \mathcal{M}_{\alpha\beta}\dot{q}_{\beta,\mathbf{k}} + \sum_{\beta} \mathcal{N}_{\alpha\beta} q_{\beta,\mathbf{k}} \ . \quad (14)$$

Here $\omega_{\alpha,k} = \sqrt{k^2 + m_\alpha^2}$ is the frequency of the mode and $\mathcal{M}$ and $\mathcal{N}$ are coupling matrices. When we quantize these variables, gravitons can be created by two effects: First, the time dependence of the effective frequency $(\omega^{\rm eff}_{\alpha,k})^2 = \omega^2_{\alpha,k} - \mathcal{N}_{\alpha\alpha}$ and second, the time dependence of the mode couplings described by the antisymmetric matrix $\mathcal{M}$ and the off-diagonal part of $\mathcal{N}$.

Note that Equation (14) is derived from the corresponding action for the variables $q_{\alpha,\mathbf{k}}$ rather than from the wave equation (7) itself. In this way the approximated boundary conditions (8) can be implemented consistently [9, 11].

In the technical paper [9] we have studied graviton production provoked by a brane moving according to (3) in great detail numerically. We have found that for long wavelengths, $kL \ll 1$, the zero mode is mainly generated by its self-coupling, i.e. the time dependence of its effective frequency. One actually finds that $\mathcal{N}_{00} \propto \delta(t)$, so that there is an instability at the moment of the kink which leads to particle creation, and the number of 4D-gravitons is given by $2v_b/(kL)^2$. This is specific to radiation dominated expansion where $H^2a^2 = -\partial_\eta(Ha)$. For another expansion law we would also obtain particle creation during the contraction and expansion phases. Light KK gravitons are produced mainly via their coupling to the zero mode. This behavior changes drastically for short wavelengths $kL \gg 1$. Then the evolution of the zero mode couples strongly to the KK modes and production of 4D gravitons via the decay of KK modes takes place. In this case the number of produced 4D gravitons decays only like $\propto 1/(kL)$.

Results and discussion: The numerical simulations have revealed a multitude of interesting effects. In the following we summarize the main findings. We refer the interested reader to Ref. [9] for an extensive discussion.

For the zero-mode power spectrum we find on scales $kL \ll 1$ on which we observe cosmological fluctuations (Mpc or larger)

$$\mathcal{P}_0(k) = \frac{\kappa_4}{2\pi^3} v_b \begin{cases} k^2 & \text{if } \ kt \ll 1 \\ \frac{1}{2}(La)^{-2} & \text{if } \ kt \gg 1 \ . \end{cases} \quad (15)$$

The spectrum of tensor perturbations is blue on super-horizon scales as one would expect for an ekpyrotic scenario. On cosmic microwave background scales the amplitude of perturbations is of the order of $(H_0/m_{\rm Pl})^2$ and hence unobservably small.

Calculating the energy density of the produced massless gravitons one obtains [9]

$$\rho_{h0} \simeq \frac{\pi}{2a^4}\frac{v_b}{LL_s^3} \ . \quad (16)$$

Comparing this with the radiation energy density, $\rho_{\rm rad} = (3/(\kappa_4 L^2))a^{-4}$, the RS fine-tuning condition leads to the simple relation

$$\rho_{h0}/\rho_{\rm rad} \simeq v_b/2. \quad (17)$$

The nucleosynthesis bound [12] requests $\rho_{h0} \lesssim 0.1\rho_{\rm rad}$, which implies $v_b \leq 0.2$, justifying our low energy approach. The model is not severely constrained by the zero-mode.

More stringent bounds come from the KK modes. Their energy density on the brane is found to be

$$\rho_{\rm KK} \simeq \frac{\pi^5}{a^6}\frac{v_b^2}{y_s}\frac{L^2}{L_s^5}. \quad (18)$$

This result is dominated by high energy KK gravitons which are produced due to the kink. It is reasonable

to require that the KK-energy density on the brane be (much) smaller than the radiation density at all times, and in particular, right after the bounce where $\rho_{\rm KK}$ is greatest. If this is not satisfied, back reaction cannot be neglected. We obtain with $\rho_{\rm rad}(0) = 3H_b^2/\kappa_4$

$$\left(\frac{\rho_{\rm KK}}{\rho_{\rm rad}}\right)\bigg|_{a=a(0)=1/\sqrt{v_b}} \simeq 100\, v_b^3 \left(\frac{L}{y_s}\right)\left(\frac{L}{L_s}\right)^2 . \qquad (19)$$

If we use the largest value for the brane velocity v_b admitted by the nucleosynthesis bound $v_b \simeq 0.2$ and require that $\rho_{\rm KK}/\rho_{\rm rad}$ be (much) smaller than one for back-reaction effects to be negligible, we obtain the very stringent condition

$$\frac{L}{y_s} < \left(\frac{L_s}{L}\right)^2 . \qquad (20)$$

Taking the largest allowed value for $L \simeq 0.1$mm, the RS fine-tuning condition Eq. (2) determines $L_s = (LL_{Pl}^2)^{1/3} \simeq 10^{-22}$mm $\simeq 1/(10^6$TeV) and $(L/L_s)^2 \simeq 10^{42}$ so that $y_s > L(L/L_s)^2 \simeq 10^{41}$mm $\sim 10^{16}$Mpc. This is about 12 orders of magnitude larger than the present Hubble scale. Also, since $y_b(t) \ll L$ in the low energy regime, and $y_s \gg L$ according to the inequality (20), the physical brane and the static brane need to be far apart at all times otherwise back reaction is not negligible. This situation is probably not very realistic. We need some high energy, stringy effects to provoke the bounce and these may well be relevant only when the branes are sufficiently close, i.e. at a distance of order L_s. But in this case the constraint (20) will be violated which implies that back reaction will be relevant. On the other hand, if we want that $y_s \simeq L$ and back reaction to be unimportant, then Eq. (19) implies that the bounce velocity has to be exceedingly small, $v_b \lesssim 10^{-15}$. One might first hope to find a way out of these conclusions by allowing the bounce to happen in the high energy regime. But then $v_b \simeq 1$ and the nucleosynthesis bound is violated since too many zero-mode gravitons are being produced. Clearly our low energy approach looses its justification if $v_b \simeq 1$, but it seems unlikely that modifications coming from the high energy regime alleviate the bounds.

Conclusions: Studying graviton production in an AdS braneworld we have found the following. First, the energy density of KK gravitons on the brane behaves as $\propto 1/a^6$, i.e. it scales like stiff matter with the expansion of the Universe and can therefore not serve as a candidate for dark matter. Furthermore, if gravity looks four dimensional on the brane, its higher-dimensional aspects, like the KK modes, are repelled from the brane. Even if KK gravitons are produced on the brane they rapidly escape into the bulk as time evolves, leaving no traces of the underlying higher-dimensional nature of gravity. This is likely to survive also in other warped braneworlds when expansion can be mimicked by brane motion.

Secondly, a braneworld bouncing at low energies is not constrained by massless 4D gravitons and satisfies the nucleosynthesis bound as long as $v_b \lesssim 0.2$. However, for interesting values of the string and AdS scales and the largest admitted bounce velocity the back reaction of the KK modes is only negligible if the two branes are far apart from each other at all times, which seems rather unrealistic. For a realistic bounce the back reaction from KK modes can most likely not be neglected. Even if the energy density of the KK gravitons on the brane dilutes rapidly after the bounce, the corresponding energy density in the bulk could even lead to important changes of the bulk geometry. The present model seems to be adequate to address the back reaction issue since the creation of KK gravitons happens exclusively at the bounce. This and the treatment of the high energy regime $v_b \simeq 1$ is reserved for future work.

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